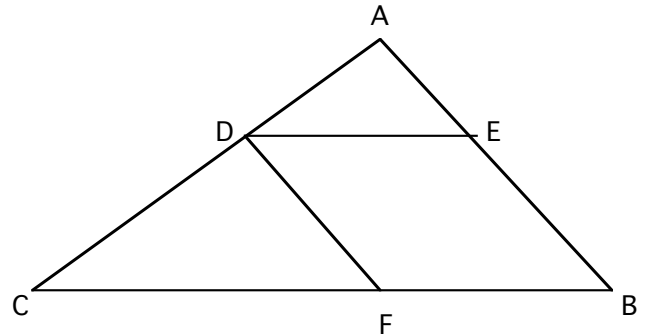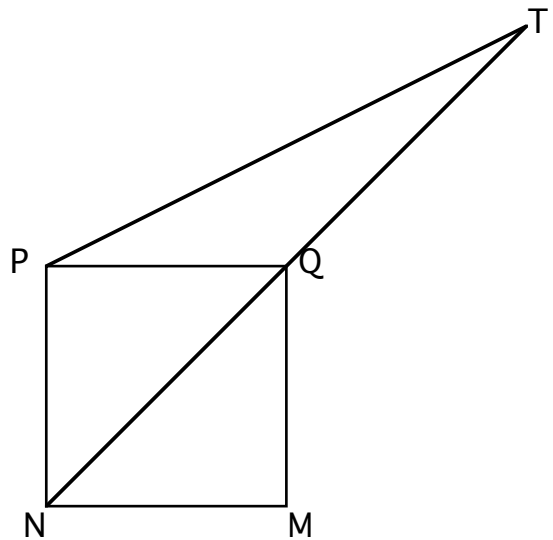Round 1:   Similarity and Pythagorean Theorem

ALL ANSWERS MUST BE IN SIMPLEST FORM

1. Given: $\triangle ABC$, $\overline{DE} \parallel \overline{CB}$, $\overline{DF} \parallel \overline{AB}$,
   $AB = 12$, $BC = 20$, $AC = 16$, and
   $AD = 4$. Find DF.

2. The area of square $MNPQ$ is 1.
   Diagonal $\overline{NQ}$ is extended its own length
   to point T. Find the length of $\overline{PT}$.

3. A median of $\triangle ABC$ divides it into two triangles of equal perimeter. The length of this
   median is 21, and the length of the side to which it is drawn is 56. What is the perimeter
   of $\triangle ABC$?

ANSWERS
(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Bancroft, South, Shepherd Hill

Round 2: Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Two hours ago, a family driving to Pennsylvania noted that the odometer reading was 45,954 miles, a palindrome, reading the same forwards and backwards. If now, when they again look at the odometer, it reads a palindrome, what, in miles per hour, has been the car's average speed?

2. Find all possible integer values of k that make <u>both</u> $x^2 + kx + 20$ and $x^2 + kx - 28$ factorable over the integers.

3. Crystallina spent her entire $ 20 weekly allowance on books and candy last week. This week she did the same, but she spent 50 % more on candy and 12.5 % less on books. How much did she spend on candy this week?

ANSWERS
(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Westboro, Quaboag, Mass Academy

Round 3:   Functions   (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $f(x) = \dfrac{x}{1-x}$, find the rule for $f(f(x))$.

2. Suppose f and g are linear functions such that $f(g(t)) = t$ for all real t.
   Find the rule for $f(x)$ if $f(5) = 3$ and $g(2) = 3$.

3. If $f(x) = \begin{cases} 2x\,, & \text{if } x < 0 \\ -1\,, & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+3\,, & \text{if } x < 0 \\ 5\,, & \text{if } x \geq 0 \end{cases}$, then the composite function

   $g(f(x)) = \begin{cases} h(x)\,, & \text{if } x < 0 \\ j(x)\,, & \text{if } x \geq 0 \end{cases}$. Write the rules for $h(x)$ and $j(x)$.

ANSWERS

(1 pt.)   1. $f(f(x)) = $ _____

(2 pts.)   2. $f(x) = $ _____

(3 pts.)   3. $h(x) = $ _____ and $j(x) = $ _____

St. John's, Auburn, Notre Dame Academy

Round 4: Combinatorics

1. The New England Patriots are in the AFC East Conference of football teams with the Jets, the Bills, and the Dolphins. How many games need to be played by pairs of teams in this conference if each team plays each other twice?

2. A mom returns from a business trip with a gift for each of her twelve children. She has three identical puzzle books, four identical coloring books, and five identical math books. In how many different ways can these gifts be distributed to her children?

3. At a Wocomal meet each team of 5 can have no more than 2 seniors and must have at least one sophomore or freshman. A coach has 20 players from which to select the first team: 6 seniors, 7 juniors, 4 sophomores, and 3 freshmen. In how many different ways can the coach form this first team?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Westborough, Assabet Valley, St. John's

Round 5:   Analytic Geometry   (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What number k makes this the equation of a single point?

$$3x^2 + 4y^2 - 6x + 8y + k = 0$$

2. The endpoints of a diameter of a circle are at $(-3, -4)$ and $(7, 2)$. Find the equation
   of this circle in the form $x^2 + y^2 = Ax + By + C$. Then write in the answer space
   the sum $A + B + C$.

3. An arch in the form of a semi-ellipse has a maximum height of p units and a span
   of $2p\sqrt{2}$ units, as shown. In terms of p, find the area of the inscribed rectangle R
   whose length is $2p$ units.
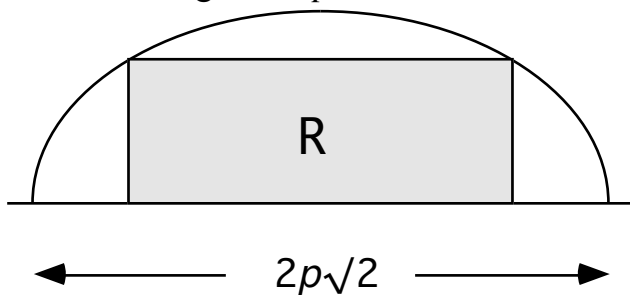
ANSWERS
(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

St. John's, Tantasqua, Quaboag

ALL ANSWERS MUST BE IN SIMPLEST FORM AND
ON THE SEPARATE TEAM ANSWER SHEET   (2 points each)

1. In rectangle $ABCD$, $CD = 8$, $AD = 4$, and E is the midpoint of $\overline{BC}$. There is a point F inside the segment $\overline{CD}$ for which $\angle AFD \cong \angle EFC$. Find length AF.

2. Of three numbers, the 3rd is one less than the sum of the other two. It is also 8 more than twice the 2nd. The 1st is one less than 3 times the difference of the 3rd minus the 2nd. What is the largest of the three numbers?

3. Find the simplified rule for $f(x)$ so that $f\left(\dfrac{x}{x+1}\right) = \dfrac{1}{x}$.

4. How many distinct three-letter "words" can you make from the letters available in BANANA ? (Assume that a letter cannot be used any more than the number of times it appears in what is given.)

5. Given $A(-3, -9)$, $B(3, -5)$, and $P(c, c)$, determine the value of c so that A, B, and P are collinear.

6. A two digit number has different digits. If the difference between the square of the number and the square of the number whose digits are interchanged is a positive perfect square, what is the two digit number?

7. Solve for x if $_xC_2 + _xC_3 = 165$.   [$_nC_r$ means "n choose r."]

8. Two circles intersect in two points. Their radii are 10 and 17, and the length of their common chord is 16. Compute the distance between their centers.

9. Determine the probability (expressed in the form $\dfrac{a}{b}$ with a and b integers) of getting at least a grade of 80 on a six-question multiple choice examination simply by guessing, if each question is equally weighted and has 4 possible answers.

Hudson, Athol, Bartlett, Westborough, Shrewsbury, Leicester, SPM, Doherty, Doherty

_____

Round 1:   Simil & Pythy

(1 pt.)   9

(2 pts.) $\sqrt{5}$ *or* 2.236

(3 pts.)   126

_____

Round 2:   Algebra 1

(1 pt.)   55
             (or accept 105 mph)

(2 pts.)   ±12

(3 pts.) $ 6

_____

Round 3:   Functions

(1 pt.) $\dfrac{x}{1-2x}$

(2 pts.) $\frac{1}{2}x + \frac{1}{2}$ *or* $\dfrac{x+1}{2}$

(3 pts.) $h(x) = 2x + 3$ *and* $j(x) = 2$

_____

_____

Round 4:   Combinatorics

(1 pt.)   12

(2 pts.)   27,720

(3 pts.)   12,712

_____

Round 5:   Ana Geom

(1 pt,)   7

(2 pts.)   31

(3 pts.) $p^2 \sqrt{2}$

_____

1. $\dfrac{20}{3}$ or $6\frac{2}{3}$ or $6.\overline{6}$ or 6.667

2. 2

3. $\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$

4. 19

5. -21

6. 65

7. 10

8. 9 or 21 (need both possibilities)

9. $\dfrac{19}{4096}$

Total Points for Team Round: _____